FAR BEYOND

MAT122

Logarithmic Function

Stony Brook University

Logarithmic Intro

Fact: logarithms and exponents are inverses of each other

use one to cancel other and/or solve

$$x = b^y \Leftrightarrow y = \log_b x$$

ex. Write in equivalent exponential form:

- $2 = \log_5 x$

2

- $3 = \log_b 64$
- $y = \log_3 7$

ex. Write in equivalent logarithmic form:

- $12^2 = x$
- $b^3 = 8$
- $e^y = 9$
- $10^y = 7$

Special Logarithmic Properties

$$\log_b b = 1$$

$$\log_b 1 = 0$$

$\ln e = 1$

$\ln 1 = 0$

Inverse Properties:

$(f \circ g)(x) = x$

$(g \circ f)(x) = x$

$$\log_b b^{x} = x$$

$$b^{\log_b x} = x$$

$\ln e^{x} = x$

$e^{\ln x} = x$

Logarithm Rules

- *condensed*
- *expanded*

Condensed Format

Product Rule $\log_b(MN)$

Quotient Rule $\log_b\left(\frac{M}{N}\right)$

Power Rule $\log_b M^P$

Product Rule

Product Rule: $\underset{\text{condensed format}}{\log_b(MN)} = \underset{\text{expanded format}}{\log_b(M) + \log_b(N)}$

ex. expand $\ln(xy)$

ex. expand $\log_4(7 \cdot 4)$

ex. expand $\log(10z)$

ex. condense $\log_3 p + \log_3 q$

ex. condense $\log_4 z + \log_4 y^7 + \log_4 5$

Quotient Rule

Quotient Rule: $\log_b\left(\frac{M}{N}\right) = \log_b(M) - \log_b(N)$

ex. expand $\log_7\left(\frac{19}{x}\right)$

ex. expand $\ln\left(\frac{e^3}{7}\right)$

ex. condense $\log_3 x^4 - \log_3 \sqrt{y}$

Power Rule

Power rule allows the exponent of the variable to become the log's coefficient and vice versa.

Power Rule: $\log_b M^P = P\log_b(M)$

ex. expand $\ln x^2$

ex. expand $\log_5 7^4$

ex. expand $\ln 3^x$

ex. expand $\ln\sqrt{x}$

$\sqrt[n]{b} = b^{1/n}$

Expand Logarithmic Expressions

ex. Expand the following as much as possible:

- $\log_b\left(x^2\sqrt{y}\right)$

- $\log_6\left(\dfrac{\sqrt[3]{x}}{36y^4}\right)$

$$\log_b(MN) = \log_b(M)+\log_b(N)$$

$$\log_b\left(\frac{M}{N}\right) = \log_b(M)-\log_b(N)$$

$$\log_b M^P = P\log_b(M)$$

Do: Expand (and evaluate where applicable):

$$\log_5\left(\frac{\sqrt{x}}{125y^3}\right)$$

Condense Logarithmic Expressions

ex. Write the following as a single logarithm:

- $\log(4x-3)+\log x$

- $3\ln(x+7)-4\ln x = \ln(x+7)^3 - \ln x^4$

- $4\log_b x + 2\log_b 6 - \frac{1}{2}\log_b y$

$$\log_b(MN) = \log_b(M)+\log_b(N)$$

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$$\log_b M^P = P\log_b(M)$$